# The High-Temperature Phases of $\mathbf{W O}_{3}$ 

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High-temperature, high-resolution neutron powder diffraction experiments were performed to investigate the phases of $\mathrm{WO}_{3}$ between room temperature and $850^{\circ} \mathrm{C}$. Two phases were found and characterized by Rietveld refinements: orthorhombic $\beta$ - $\mathrm{WO}_{3}$ (Pbcn, $a=7.3331(2), b=7.5733(2), c=7.7401(3) \AA$ at $350^{\circ} \mathrm{C}$, tilt system $a^{0} b^{+} c^{-}$) and tetragonal $\alpha-\mathrm{WO}_{3}(P 4 / n c c, a=$ $5.2759(1), b=5.2759(1), c=7.8462(3) \AA$ at $800^{\circ} \mathrm{C}$, tilt system $a^{0} a^{0} c^{-}$). The sequence of temperature-induced phase transitions in $\mathrm{WO}_{3}$ can be rationalized in terms of changes in the octahedral tilt systems and/or displacements of the tungsten out of the center of the $\mathrm{WO}_{6}$ octahedron. Above room temperature the two phase transitions are driven by sucessive softening of phonon modes, $M_{3}$ at the $\alpha$ - to $\beta$-transition and $R_{25}$ at the $\beta$ - to $\gamma$ transition. © 1999 Academic Press

## INTRODUCTION

Despite its simple stoichiometry, the structural distortions and phase transitions of $\mathrm{WO}_{3}$ are quite complex. All of the polymorphs of $\mathrm{WO}_{3}$ can be described as distortions from the cubic $\mathrm{ReO}_{3}$ structure, which consists of threedimensional network of corner-sharing $M \mathrm{O}_{6}$ octahedra. The cubic perovskite structure, $A M \mathrm{O}_{3}$, is closely related. It can be generated from the $\mathrm{ReO}_{3}$ structure by insertion of cations ( $A$ site) into the cuboctahedral vacancies present in the $\mathrm{ReO}_{3}$ framework (Fig. 1). Thus, both structures have the same topology and not surprisingly undergo similar distortions from the ideal cubic structure. Predominant distortion mechanisms include tilting of the $M \mathrm{O}_{6}$ octahedra (1-3), displacements of the $M$ cation from the center of the octahedron (4), and distortions of the octahedra $(5,6)$. Over the past decade there has been a considerable amount of work carried out toward the goal of elucidating the intricate structural distortions of perovskites, due to their fascinating physical properties such as superconductivity (7-9), ferroelectricity (10), and giant magnetoresistance (11, 12). Since $\mathrm{WO}_{3}$ undergoes at least four phase transitions, each one
resulting from a change in the octahedral tilting and/or cooperative tungsten shifts, it is an ideal compound for studying the behaviour and evolution of structural distortions in the $\mathrm{ReO}_{3} /$ perovskite framework.
The structure of $\mathrm{WO}_{3}$ is reported in the literature to progress through the following series of phase transitions upon heating: monoclinic $\left(\varepsilon-\mathrm{WO}_{3}\right) \rightarrow$ triclinic $\left(\delta-\mathrm{WO}_{3}\right) \rightarrow$ monoclinic $\left(\gamma-\mathrm{WO}_{3}\right) \rightarrow$ orthorhombic $\left(\beta-\mathrm{WO}_{3}\right) \rightarrow$ tetragonal $\left(\alpha-\mathrm{WO}_{3}\right)(13)$. Most of the early structural work done on $\mathrm{WO}_{3}$ are based on single-crystal X-ray diffraction measurements. There are two inherent complications to this approach. First, accurate determination of oxygen positions is difficult due to the fact that the scattering power of tungsten is much larger than that of oxygen. Second, the high degree of pseudo-symmetry present and the large number of phase transitions encountered upon cooling typically lead to severe problems with twinning. Both of these complications can be minimized by the use of high-resolution neutron powder diffraction methods. The neutron scattering lengths of tungsten $(b=4.86(2) \mathrm{fm})$ and oxygen $(b=5.803(4) \mathrm{fm})$ (14) are comparable, and the use of powder techniques minimizes complications arising from twinning. The advantages of this approach were demonstrated in a relatively recent study of $\delta-\mathrm{WO}_{3}$ and $\gamma-\mathrm{WO}_{3}$ (15), which resulted in more reliable oxygen positional parameters with respect to earlier studies using low resolution neutron powder ( $\gamma$ $\left.\mathrm{WO}_{3}\right)(16)$ and single-crystal X-ray $\left(\delta-\mathrm{WO}_{3}\right)(17)$ diffraction methods. More recently, the structure of the low-temperature phase, $\varepsilon$ - $\mathrm{WO}_{3}$, was determined independently by two groups, both using high-resolution neutron powder diffraction techniques $(13,18)$. The structure is noncentrosymmetric and monoclinic with ion shifts which confirm earlier evidence of ferroelectric behavior (19). Interestingly, another recent study found the room temperature, high pressure phase of $\mathrm{WO}_{3}$ to exhibit a crystal structure similar to that of $\varepsilon$ - $\mathrm{WO}_{3}$. Despite this recent flurry of $\mathrm{WO}_{3}$ structural studies, the crystallographic data available above room temperature can best be described as approximate. We report here the


FIG. 1. Perovskite structure $A M \mathrm{O}_{3}$ showing the three-dimensional corner sharing $\mathrm{ReO}_{3}$ network of $M \mathrm{O}_{6}$ octahedra and the cuboctahedral $A$ site.
results of a high-resolution neutron powder diffraction study of $\mathrm{WO}_{3}$ over the temperature range $300-850^{\circ} \mathrm{C}$.

## EXPERIMENTAL

The sample was the same one that was used in our earlier studies $(13,15)$. Briefly recounting its preparation, reagent grade $\mathrm{WO}_{3}$ was heated in air at $1200^{\circ} \mathrm{C}$ for 8 h , cooled at a rate of $20^{\circ} \mathrm{C} / \mathrm{h}$ to $800^{\circ} \mathrm{C}$, and then cooled at $100^{\circ} \mathrm{C} / \mathrm{h}$ to room temperature. The neutron powder diffraction patterns were measured at the HIFAR research reactor, operated by the Australian Nuclear Science and Technology Organization (ANSTO) at Lucas Heights. We used the high-resolution powder diffractometer (20), which has a 24 bank detector and provides neutrons with a wavelength of $1.4932(2)(\AA)$. Data were collected using $\sim 10 \mathrm{~g}$ of polycrystalline $\mathrm{WO}_{3}$, over an angular range of $0.5^{\circ}<2 \theta<154^{\circ}$ at seven different temperatures: $300,400,550,700,800$, and $850^{\circ} \mathrm{C}$. The sample was held in a thin-walled $12-\mathrm{mm}$ diameter stainless steel can, and the heated sample chamber was open to the atmosphere. The Rietveld refinements were
performed using the software package PROFIL (written by J. K. Cockcroft) (21). The background was determined by a linear interpolation between points without Bragg contributions. A pseudo-Voigt profile function was used to describe the peak shape. Two peaks that resulted from the furnace were excluded and the steel can was refined as $\alpha-\mathrm{Fe}$. The weighted profile residual used in PROFIL is defined as $R_{\mathrm{wp}}=\left(\Sigma\left[\mathrm{w}\left(Y_{\mathrm{obs}}-Y_{\mathrm{calc}}\right)^{2} / \Sigma\left[\mathrm{w}\left(Y_{\mathrm{obs}}\right)^{2}\right]\right)^{1 / 2}\right.$, where the background has been subtracted from observed $y_{\text {total }}$ to yield the intensity $Y_{\text {obs }}$ and the weight is defined as $w=$ $1 /\left[y_{\text {total }}+\right.$ background $]$. $\Sigma$ refers only to parts of the pattern containing Bragg peaks. $R_{I}$ is defined as $R_{I}=\left(\Sigma\left|\left\{I_{\text {obs }}-I_{\text {calc }}\right\}\right| / \Sigma I_{\text {obs }}\right) \quad$ and $\quad R_{\text {exp }}=([N-P+C] /$ $\left.\Sigma\left[\mathrm{w}\left(Y_{\mathrm{obs}}\right)^{2}\right]\right)^{1 / 2}$ with $N$ being the number of contributing observations, $P$ is the number of contributing reflections, and $C$ is the number of constraints.

## RESULTS

Orthorhombic $\beta-W_{3}$
Previous X-ray diffraction studies on a single crystal of $\mathrm{WO}_{3}$ twinned along [100] led Salje to conclude that $\mathrm{WO}_{3}$ was orthorhombic with space group symmetry Pmnb, from approximately 320 to $720^{\circ} \mathrm{C}$ (22). This structure served as the starting point for out initial refinement attempts. Somewhat surprisingly, we found that starting from this model we could not obtain reasonable fits to the data. A careful examination of the extinction conditions showed them to be consistent with space group Pcnb. Pcnb is a nonstandard setting of space group Pbcn (60), but is chosen for this work to maintain consistency with the monoclinic $(\gamma)$ and tetragonal $(\alpha)$ phases. Since the extinction conditions associated with Pcnb are unique, we focussed out efforts on structural models belonging to this space group. We were able to quickly obtain a model which led to a good fit to the experimental diffraction pattern. Figure 2 shows a fit of this model to the data recorded at $550^{\circ} \mathrm{C}$. Table 1 lists the Rietveld refinement results obtained at 350,400 , 550, and $700^{\circ} \mathrm{C}$. Figure 3 shows the evolution of the cell constants between room temperature and $850^{\circ} \mathrm{C}$. The first data point was taken from our previous work (15), the point at $300^{\circ} \mathrm{C}$ is the first point from this study. One notes that while the $a$ and $c$ axes expand as expected between 350 and $700^{\circ} \mathrm{C}$, the $b$ axis shows no comparable thermal expansion rather a very slight contraction between 400 and $700^{\circ} \mathrm{C}$. We will return to this point in the Discussion.

## Tetragonal $\alpha-\mathrm{WO}_{3}$

Early work by Kehl et al. (12) describes the structure of $\mathrm{WO}_{3}$ above $720^{\circ} \mathrm{C}$ as being tetragonal with space group $P 4 / \mathrm{nmm}$ and lattice parameters, $a=5.272(2)$ and $c=$ $3.920(2) \AA(23)$. Although we tried Rietveld refinements using this structure as the starting point, it was quickly evident


FIG. 2. Rietveld refinement plot of orthorhombic $\beta-\mathrm{WO}_{3}$ at $550^{\circ} \mathrm{C}$. The bottom curve depicts the difference between experimental observations (points) and intensities calculated and refined to the crystallographic model. The tick marks indicate the positions of the Bragg reflections in $2 \theta$. The horizontal bars indicate regions contaminated with scattering from the furnace shielding which were excluded from the refinement.
that such a model could not adequately fit the data. An examination of the peak positions clearly showed that the $c$ axis was doubled, with respect to Kehl's model, and that the extinction conditions were consistent with space group $P 4 / n c c$. As with the orthorhombic phase we were once again fortunate that the extinction conditions could be used to uniquely identify the space group. Since the basic structural framework was already known it was relatively straightforward to construct a model structure with $P 4 / n c c$ symmetry. Figure 4 shows a fit to the data collected at $850^{\circ} \mathrm{C}$ using this model and Table 1 lists the structural parameters from refinements at 800 and $850^{\circ} \mathrm{C}$. In contrast to the phase transition between the monoclinic $(\gamma)$ and orthorhombic $(\beta)$ phases, which appears to be a second (or higher) order phase transition, the $\beta$ - to $\alpha-\mathrm{WO}_{3}$ transition is first order with marked discontinuities in the lattice parameters and bond distances.

## DISCUSSION

It is instructive to rationalize the phase transitions of $\mathrm{WO}_{3}$ with respect to the possible $\mathrm{ReO}_{3} /$ perovskite distortion mechanisms: distortions of the octahedra, tilting of the octahedra, and tungsten displacements. To aid in this analysis the program IVTON (24) was used to calculate the volume of the $\mathrm{WO}_{6}$ octahedra and the fractional coordinates of the octahedral center. For the sake of discussion we will insert a virtual (and nonexistent) atom, $\mathrm{W}_{\mathrm{c}}$, at the center of the octahedron. Furthermore, the oxygen ions which are
displaced from tungsten primarily in the $a$ direction will be denoted $\mathrm{O}_{x}$, while those displaced predominantly in the $b$ and $c$ directions will be referred to as $\mathrm{O}_{y}$ and $\mathrm{O}_{z}$, respectively (25).
Table 2 contains the results of the IVTON analysis for each of the five known phases of $\mathrm{WO}_{3}$. To assess the degree of octahedral distortion in each phase we examined the $\mathrm{W}_{\mathrm{c}}-\mathrm{O}$ distances. Because $\mathrm{W}_{\mathrm{c}}$ sits at the center of the octahedron, all three $\mathrm{W}_{\mathrm{c}}-\mathrm{O}$ distances will be identical if the octahedron is undistorted. $\mathrm{W}_{\mathrm{c}}-\mathrm{O}$ distances greater than the average $\mathrm{W}_{\mathrm{c}}-\mathrm{O}$ distance indicate elongation of the octahedra in that direction, while values smaller than the average indicate compression. The first piece of information we can gain from examination of the $\mathrm{W}_{\mathrm{c}}-\mathrm{O}$ distances in Table 2 is that the octahedral distortions are rather small when compared with the W displacements. There is however, a moderate elongation of the octahedra in the $c$ direction. This is due to the extreme long-short-long-short bond distance alternation along the $c$ direction. This distortion of the octahedra is relatively constant from 15 to 1123 K , emphasizing the fact that octahedral distortions do not play an important role in tungsten trioxide's complex sequence of phase transitions.

Examination of the $\mathrm{W}-\mathrm{W}_{\mathrm{c}}$ distances sheds light on the magnitude and direction of the tungsten shifts. In each of the five phases the shift in the $c$ direction is considerably larger than the shift in the $a b$ plane. This is responsible for elongation of the octahedra in the $c$ direction, as well as the fact that the long-short $\mathrm{W}-\mathrm{O}$ bond distance alternation is most extreme in the $c$ direction, and the marked expansion of the $c$ lattice parameter, with respect to the $a$ and $b$ lattice parameters. Considering the tungsten shifts in the $a b$ plane separately, we see that in $\varepsilon$ - $\mathrm{WO}_{3}$ the shift is primarily toward an edge of the octahedron (in the $a b$ plane), in $\delta-, \gamma$, and $\beta-\mathrm{WO}_{3}$ the shift is predominantly toward a corner of the octahedron (in the $a b$ plane), and in $\alpha-\mathrm{WO}_{3}$ there is no component of the displacement in the $a b$ plane. The $\mathrm{W}-\mathrm{W}_{\mathrm{c}}$ and $\mathrm{W}-\mathrm{O}$ distances both support the conclusion that abrupt changes in directionality of the tungsten shifts occur only at the $\varepsilon$ - to $\delta$ - and $\beta$ - to $\alpha-\mathrm{WO}_{3}$ phase transitions. Abrupt changes in the lattice parameters leave little doubt that both of these phase transitions are first order. In this regard $\mathrm{WO}_{3}$ behaves in a fashion similar to $\mathrm{BaTiO}_{3}$, which shows a first-order phase transition each time the directionality of the titanium shift changes (26).

Finally, changes in octahedral tilting can easily be followed by examination of the Glazer tilt system. Apart from the $\varepsilon$ - to $\delta-\mathrm{WO}_{3}$ phase transition, the other three transitions are characterized by changes in the octahedral tilt system. The tilt systems for $\alpha$ - and $\beta-\mathrm{WO}_{3}$ are illustrated in Fig. 5. In these drawings the in-phase, out-of-phase, and absence of tilting about each of the Cartesian axes can clearly be seen. Similar illustrations for the other phases are presented in our previous publications $(13,15)$. The tilt system evolution

TABLE 1
Structural Parameters of $\mathrm{WO}_{3}$ between 300 and $850^{\circ} \mathrm{C}$
$T=300^{\circ} \mathrm{C} \quad$ space group $P 2_{1} / n \quad a=7.3271(2) \AA \quad b=7.5644(2) \AA \quad c=7.7274(3) \AA \quad \beta=90.488(3)^{\circ}$
$R_{\mathrm{wp}}=14.0 \% \quad R(\mathrm{I})=8.8 \% \quad R_{\text {exp }}=6.9 \% \quad 1682$ reflections $\quad \chi^{2}=4.13$

| Atom | $x$ | $y$ | $z$ | $B(1,1)$ | $B(2,2)$ | $B(3,3)$ | $B(2,3)$ | $B(3,1)$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | ---: | ---: | ---: |
| $\mathrm{W}(1)$ | $0.253(1)$ | $0.026(1)$ | $0.283(1)$ | $1.0(1)$ | $1.0(1)$ | $1.0(1)$ | $1.0(1)$ | $1.0(1)$ | $0.8(1)$ |
| $\mathrm{W}(2)$ | $0.246(1)$ | $0.033(1)$ | $0.7811(9)$ | $0.8(1)$ | $0.8(1)$ | $0.8(1)$ | $0.8(1)$ | $0.8(1)$ |  |
| $\mathrm{O}(1)$ | $0.000(1)$ | $0.030(1)$ | $0.218(1)$ | $0.7(3)$ | $1.5(4)$ | $3.0(5)$ | $0.5(3)$ | $-0.4(3)$ | $0.8(3)$ |
| $\mathrm{O}(2)$ | $1.002(1)$ | $0.464(1)$ | $0.221(1)$ | $1.8(4)$ | $1.0(4)$ | $2.3(5)$ | $0.1(3)$ | $1.0(3)$ | $0.2(3)$ |
| $\mathrm{O}(3)$ | $0.282(1)$ | $0.264(2)$ | $0.277(1)$ | $2.0(3)$ | $2.5(4)$ | $2.6(4)$ | $1.9(4)$ | $0.8(3)$ | $-0.3(4)$ |
| $\mathrm{O}(4)$ | $0.2140(9)$ | $0.257(1)$ | $0.742(1)$ | $2.1(3)$ | $0.1(3)$ | $3.5(4)$ | $-0.1(3)$ | $0.4(3)$ | $-0.1(3)$ |
| $\mathrm{O}(5)$ | $0.277(1)$ | $0.028(1)$ | $0.0002(9)$ | $1.6(3)$ | $2.5(3)$ | $1.0(3)$ | $0.3(3)$ | $-0.6(2)$ | $0.9(3)$ |
| $\mathrm{O}(6)$ | $0.288(1)$ | $0.498(1)$ | $1.000(1)$ | $1.6(3)$ | $2.0(4)$ | $3.6(4)$ | $0.8(3)$ | $1.0(3)$ | $0.1(2)$ |

$T=350^{\circ} \mathrm{C} \quad$ space group Pcnb $\quad a=7.3331(2) \AA \quad b=7.5733(2) \AA \quad c=7.7401(3) \AA$
$R_{\mathrm{wp}}=14.2 \% \quad R(\mathrm{I})=9.4 \% \quad R_{\exp }=6.5 \% \quad 476$ reflections $\quad \chi^{2}=4.72$

| Atom | $x$ | $y$ | $z$ | $B(1,1)$ | $\mathrm{B}(2,2)$ | $\mathrm{B}(3,3)$ | $\mathrm{B}(2,3)$ | $\mathrm{B}(3,1)$ | $\mathrm{B}(1,2)$ |
| :--- | ---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{W}(1)$ | $0.2521(6)$ | $0.0291(6)$ | $0.2830(5)$ | $0.72(6)$ | $0.72(6)$ | $0.72(6)$ | $0.72(6)$ | $0.72(6)$ | $0.72(6)$ |
| $\mathrm{O}(1)$ | $-0.0016(9)$ | $0.0322(4)$ | $0.2205(5)$ | $0.5(1)$ | $1.3(1)$ | $2.7(2)$ | $0.2(1)$ | $-0.8(2)$ | $0.2(2)$ |
| $\mathrm{O}(2)$ | $0.2833(5)$ | $0.2693(6)$ | $0.2592(7)$ | $1.6(1)$ | $0.6(1)$ | $3.3(2)$ | $0.7(2)$ | $-0.2(2)$ | $0.1(2)$ |
| $\mathrm{O}(3)$ | $0.2803(5)$ | $0.0130(7)$ | $0.0017(6)$ | $1.4(1)$ | $2.6(2)$ | $1.5(2)$ | $0.1(1)$ | $-0.1(2)$ | $-0.1(2)$ |

$T=400^{\circ} \mathrm{C} \quad$ space group Pcnb $\quad a=7.3397(2) \AA \quad b=7.5744(2) \AA \quad c=7.7452(3) \AA$
$R_{\mathrm{wp}}=13.1 \% \quad R(\mathrm{I})=8.7 \% \quad R_{\text {exp }}=6.6 \% \quad 477$ reflections $\quad \chi^{2}=3.99$

| Atom | $x$ | $y$ | $z$ | $\mathrm{~B}(1,1)$ | $\mathrm{B}(2,2)$ | $\mathrm{B}(3,3)$ | $\mathrm{B}(2,3)$ | $\mathrm{B}(3,1)$ |  |
| :--- | ---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{W}(1)$ | $0.2520(5)$ | $0.0291(5)$ | $0.2830(4)$ | $0.73(6)$ | $0.73(6)$ | $0.73(6)$ | $0.73(6)$ | $0.73(6)$ | $0.73(6)$ |
| $\mathrm{O}(1)$ | $-0.0016(8)$ | $0.0322(4)$ | $0.2205(5)$ | $0.47(9)$ | $1.3(1)$ | $2.7(2)$ | $0.2(1)$ | $-0.8(2)$ | $0.2(2)$ |
| $\mathrm{O}(2)$ | $0.2833(4)$ | $0.2582(5)$ | $0.2591(6)$ | $1.6(1)$ | $0.6(1)$ | $3.4(2)$ | $0.7(2)$ | $-0.2(1)$ | $0.1(2)$ |
| $\mathrm{O}(3)$ | $0.2803(4)$ | $0.0130(7)$ | $0.0016(5)$ | $1.4(1)$ | $2.6(2)$ | $1.5(2)$ | $0.1(1)$ | $-0.1(2)$ | $-0.1(2)$ |

$$
\begin{gathered}
T=550^{\circ} \mathrm{C} \quad \text { space group } P c n b \quad a=7.3612(2) \AA \\
R_{\mathrm{wp}}=12.3 \%
\end{gathered} \quad R(\mathrm{I})=8.2 \% \quad R_{\text {exp }}=6.7 \% \quad 4.5739(2) \AA \quad c=7.7620(3) \AA \begin{array}{lll}
\text { reflections } & \chi^{2}=3.36
\end{array}
$$

| Atom | $x$ | $y$ | $z$ | $B(1,1)$ | $\mathrm{B}(2,2)$ | $\mathrm{B}(3,3)$ | $\mathrm{B}(2,3)$ | $\mathrm{B}(3,1)$ | $\mathrm{B}(1,2)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{W}(1)$ | $0.2523(6)$ | $0.0294(5)$ | $0.2834(4)$ | $0.81(6)$ | $0.81(6)$ | $0.81(6)$ | $0.81(6)$ | $0.81(6)$ | $0.81(6)$ |
| $\mathrm{O}(1)$ | $0.0017(9)$ | $0.0300(4)$ | $0.2231(5)$ | $0.47(9)$ | $1.5(1)$ | $2.9(2)$ | $0.1(1)$ | $-0.5(2)$ |  |
| $\mathrm{O}(2)$ | $0.2816(4)$ | $0.2607(5)$ | $0.2579(7)$ | $2.0(2)$ | $0.4(1)$ | $4.2(2)$ | $0.5(2)$ | $-0.3(2)$ | $0.2(2)$ |
| $\mathrm{O}(3)$ | $0.2768(5)$ | $0.0128(7)$ | $0.0027(5)$ | $2.0(2)$ | $2.9(2)$ | $1.4(1)$ | $0.1(1)$ | $-0.3(2)$ | $0.0(2)$ |

$T=700^{\circ} \mathrm{C} \quad$ space group Pcnb $\quad a=7.3903(2) \AA \quad b=7.5655(3) \AA \quad c=7.7875(3) \AA$ $R_{\mathrm{wp}}=12.8 \% \quad R(\mathrm{I})=9.2 \% \quad R_{\exp }=6.7 \% \quad 485$ reflections $\quad \chi^{2}=3.65$

| Atom | $x$ | $y$ | $z$ | $B(1,1)$ | $\mathrm{B}(2,2)$ | $\mathrm{B}(3,3)$ | $\mathrm{B}(2,3)$ | $\mathrm{B}(3,1)$ | $\mathrm{B}(1,2)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{W}(1)$ | $0.2502(8)$ | $0.0265(7)$ | $0.2839(4)$ | $0.99(8)$ | $0.99(8)$ | $0.99(8)$ | $0.99(8)$ | $0.99(8)$ | $0.99(8)$ |
| $\mathrm{O}(1)$ | $0.000(1)$ | $0.0280(5)$ | $0.2293(8)$ | $0.5(1)$ | $1.9(1)$ | $3.5(2)$ | $0.2(2)$ | $-0.8(2)$ | $0.4(2)$ |
| $\mathrm{O}(2)$ | $0.2782(5)$ | $0.2615(7)$ | $0.258(1)$ | $2.1(2)$ | $0.6(1)$ | $4.5(2)$ | $-0.1(2)$ | $-0.5(2)$ | $0.2(2)$ |
| $\mathrm{O}(3)$ | $0.2694(8)$ | $0.0086(9)$ | $0.0039(6)$ | $2.6(2)$ | $3.7(2)$ | $1.6(2)$ | $0.3(2)$ | $0.4(3)$ | $-1.1(5)$ |

$$
T=800^{\circ} \mathrm{C} \quad \text { space group } P 4 / n c c \quad a=5.2759(1) \AA \quad b=5.2759(1) \AA \quad c=7.8462(3) \AA
$$

$$
R_{\mathrm{wp}}=10.4 \% \quad R(\mathrm{I})=6.7 \% \quad R_{\mathrm{exp}}=5.0 \% \quad 132 \text { reflections } \quad \chi^{2}=4.32
$$

| Atom | $x$ | $y$ | $z$ | $\mathrm{~B}(1,1)$ | $\mathrm{B}(2,2)$ | $\mathrm{B}(3,3)$ | $\mathrm{B}(2,3)$ | $\mathrm{B}(3,1)$ | $\mathrm{B}(1,2)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: | :---: |
| $\mathrm{W}(1)$ | $1 / 4$ | $1 / 4$ | $0.2847(4)$ | $1.37(8)$ | $1.37(8)$ | $2.0(1)$ | 0 | 0 | 0 |
| $\mathrm{O}(1)$ | $1 / 4$ | $1 / 4$ | $0.0028(4)$ | $4.3(1)$ | $4.3(1)$ | $1.3(1)$ | 0 | 0 | 0 |
| $\mathrm{O}(2)$ | $0.0287(2)$ | $-0.0287(2)$ | $1 / 4$ | $1.75(5)$ | $1.75(5)$ | $5.9(2)$ | $-0.22(9)$ | $-0.22(9)$ | $-0.83(7)$ |

TABLE 1—Continued

| $\begin{gathered} T=850^{\circ} \mathrm{C} \quad \text { space group } P 4 / n c c \quad a=5.2806(1) \AA \quad b=5.2806(1) \AA \quad c=7.8496(3) \AA \\ R_{\mathrm{wp}}=11.7 \% \quad R(\mathrm{I})=7.1 \% \quad R_{\exp }=4.8 \% \quad \text { 131 reflections } \quad \chi^{2}=6.08 \end{gathered}$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Atom | $x$ | $y$ | $z$ | $\mathrm{B}(1,1)$ | B $(2,2)$ | B $(3,3)$ | B $(2,3)$ | B $(3,1)$ | $\mathrm{B}(1,2)$ |
| W(1) | 1/4 | 1/4 | 0.2849(5) | 1.38(9) | 1.38(9) | 2.5(2) | 0 | 0 | 0 |
| $\mathrm{O}(1)$ | 1/4 | 1/4 | 0.0025(5) | 4.5(1) | 4.5(1) | 1.4(2) | 0 | 0 | 0 |
| O(2) | 0.0262(3) | -0.0262(3) | 1/4 | 1.94(6) | 1.94(6) | 6.0(2) | -0.2(1) | -0.2(1) | -0.93(8) |

from tetragonal $\alpha-\mathrm{WO}_{3}\left(a^{0} a^{0} c^{-}\right)$to orthorhombic $\beta-\mathrm{WO}_{3}$ $\left(a^{0} b^{+} c^{-}\right)$and finally to monoclinic $\gamma-\mathrm{WO}_{3}\left(a^{-} b^{+} c^{-}\right)$ is analogous to the sequence of phase transitions observed in $\mathrm{SrZrO}_{3}$ (27). These phase transitions are driven by a
successive freezing out of soft phonon modes, $M_{3}$ at the $\alpha$ - to $\beta$-transition and $R_{25}$ at the $\beta$ - to $\gamma$ - transition. Roth and Waring (28) discussed the possibility of an additional phase transition above $900^{\circ} \mathrm{C}$, where the $c$ axis goes from

TABLE 2

| Phase | $\varepsilon^{1,2}$ | $\delta^{1}$ | $\gamma^{1}$ | $\beta$ | $\alpha^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Temperature (K) | 15 | 298 | 573 | 823 | 1123 |
| Crystal System | Monoclinic | Triclinic | Monoclinic | Orthorhombic | Tetragonal |
| $a(\AA)$ | 7.378 | 7.309 | 7.327 | 7.361 | 7.468 |
| $b(\AA)$ | 7.378 | 7.522 | 7.564 | 7.574 | 7.468 |
| $c(\AA)$ | 7.664 | 7.686 | 7.727 | 7.762 | 7.850 |
| $\alpha\left({ }^{\circ}\right)$ | 88.73 | 89.85 | 90 | 90 | 90 |
| $\beta\left({ }^{\circ}\right)$ | 91.27 | 90.91 | 90.49 | 90 | 90 |
| $\gamma\left({ }^{\circ}\right)$ | 91.34 | 90.94 | 90 | 90 | 90 |
| Volume ( $\AA^{3}$ ) | 417.0 | 422.5 | 428.3 | 432.7 | 437.8 |
| Tilt system | $a^{-} b^{-} c^{-}$ | $a^{-} b^{-} c^{-}$ | $a^{-} b^{+} c^{-}$ | $a^{0} b^{+} c^{-}$ | $a^{0} b^{0} c^{-}$ |
| Octahedral Vol. ( $\AA^{3}$ ) | 9.32 | 9.31 | 9.27 | 9.26 | 9.22 |
| Distance W-W $\mathrm{W}_{\mathrm{c}}(\AA)$ |  |  |  |  |  |
| $x$ | 0.06 | 0.03 | 0.02 | 0.00 | 0 |
| $y$ | 0.10 | 0.17 | 0.14 | 0.14 | 0 |
| $z$ | 0.27 | 0.24 | 0.24 | 0.24 | 0.25 |
| Total | 0.30 | 0.29 | 0.29 | 0.28 | 0.25 |
| $\mathrm{W}_{\mathrm{c}}-\mathrm{O}$ distances $(\AA)$ |  |  |  |  |  |
| $\mathrm{W}_{\mathrm{c}}-\mathrm{O}_{x}$ | 1.89 | 1.87 | 1.87 | 1.87 | 1.88 |
| $\mathrm{W}_{\mathrm{c}}-\mathrm{O}_{y}$ | 1.89 | 1.91 | 1.91 | 1.91 | 1.88 |
| $\mathrm{W}_{\mathrm{c}}-\mathrm{O}_{z}$ | 1.96 | 1.95 | 1.95 | 1.95 | 1.96 |
| Average | 1.91 | 1.91 | 1.91 | 1.91 | 1.91 |
| W-O distances ( $\AA$ ) |  |  |  |  |  |
| $\mathrm{W}-\mathrm{O}_{x}$ | 1.81/2.02 | 1.84/1.96 | 1.86/1.93 | 1.89/1.90 | 1.90 |
| $\mathrm{W}-\mathrm{O}_{y}$ | 1.80/2.02 | 1.78/2.08 | 1.78/2.08 | 1.78/2.08 | 1.90 |
| $\mathrm{W}-\mathrm{O}_{z}$ | 1.75/2.18 | 1.75/2.17 | 1.71/2.20 | 1.72/2.19 | 1.71/2.22 |
| Average | 1.93 | 1.93 | 1.93 | 1.93 | 1.92 |
| $\mathrm{W}_{\mathrm{c}}-\mathrm{O}-\mathrm{W}_{\mathrm{c}}$ bond angles ( ${ }^{\circ}$ ) |  |  |  |  |  |
| $\mathrm{W}_{\mathrm{c}}-\mathrm{O}_{x}-\mathrm{W}_{\mathrm{c}}$ | 153.9 | 156.2 | 157.9 | 160.9 | 168.0 |
| $\mathrm{W}_{\mathrm{c}}-\mathrm{O}_{y}-\mathrm{W}_{\mathrm{c}}$ | 155.6 | 159.1 | 162.5 | 165.6 | 168.0 |
| $\mathrm{W}_{\mathrm{c}}-\mathrm{O}_{z}-\mathrm{W}_{\mathrm{c}}$ | 156.9 | 159.4 | 164.0 | 168.4 | 180.0 |
| Bond valences |  |  |  |  |  |
| W | 6.38 | 6.37 | 6.53 | 6.48 | 6.50 |
| $\mathrm{O}_{x}$ | 2.13 | 2.15 | 2.17 | 2.13 | 2.13 |
| $\mathrm{O}_{y}$ | 2.14 | 2.12 | 2.14 | 2.14 | 2.13 |
| $\mathrm{O}_{z}$ | 2.11 | 2.09 | 2.23 | 2.21 | 2.23 |

[^0]

FIG. 3. The evolution of the cell parameters of the $\gamma$-, $\beta$-, and $\alpha$-phase of $\mathrm{WO}_{3}$ with temperature.
approximately 7.8 to 3.9 Å. Such a transition would presumably be driven by elimination of the last octahedral tilt, about the $c$ axis. The structural evolution of $\mathrm{WO}_{3}$ above room temperature would then be completely analogous to that observed for $\mathrm{SrZrO}_{3}$. Experiments are currently underway


FIG. 4. Rietveld refinement plot of orthorhombic $\alpha-\mathrm{WO}_{3}$ at $850^{\circ} \mathrm{C}$. The bottom curve depicts the difference between experimental observations (points) and intensities calculated and refined to the crystallographic model. The tick marks indicate the positions of the Bragg reflections in $2 \theta$. The horizontal bars indicate regions contaminated with scattering from the furnace shielding which were excluded from the refinement.
a

d

b

c

e

f


FIG. 5. Polyhedral representations of the structure of orthorhombic $\beta$ $\mathrm{WO}_{3}$ looking down the (a) [100], (b) [010], and (c) [001] directions, and tetragonal $\alpha-\mathrm{WO}_{3}$ looking down the (d) [100], (e) [110], and (f) [001] directions.
to investigate this possibility. It is also interesting to note that according to the recent work of Howard and Stokes (3), a second-order phase transition is allowed for the orthogonal $(\beta)$ to monoclinic $(\gamma)$ transition, but not for the monoclinic $(\gamma)$ to triclinic $(\delta)$ transition. This prediction is consistent with the experimental observations, which suggest that the $\beta$ to $\gamma$ transition is the only second-order phase transition of $\mathrm{WO}_{3}$ thus far observed.

## CONCLUSION

All four phase transitions of $\mathrm{WO}_{3}$ can be described in terms of changes in the octahedral tilt system and/or changes in the displacement of tungsten away from the center of its octahedron. Changes in the octahedral tilt system accompany the $\delta$ - to $\gamma$-, $\gamma$ - to $\beta$-, and $\beta$ - to $\alpha$-transitions, while abrupt changes in the directionality of the tungsten displacements accompany the $\varepsilon$ - to $\delta$ - and $\alpha$ - and $\beta$-transitions. The tungsten displacements can be approximately described as toward a corner of the octahedron in $\alpha-\mathrm{WO}_{3}$, toward an edge in $\beta-\gamma-$, and $\delta-\mathrm{WO}_{3}$, and toward a face of the octahedron in $\varepsilon$ - $\mathrm{WO}_{3}$. Having said that it is important to note that
for each phase the $c$ component of the tungsten displacement is considerably larger than the component in the $a b$ plane.

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[^0]:    ${ }^{1}$ The $\varepsilon$ - and $\delta$-phases contain 2 W and 6 O sites, while the $\delta$-phase contains 4 W and 12 O sites per unit cell. For these phases the values in the table were obtained by averaging over the appropriate sites.
    ${ }^{2}$ The large $Z=8$ cell has been used for $\varepsilon$ - and $\alpha$ - phases for comparison with the other polymorphs.

